Assignment 7

5-2. The system

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -x_1(t) + [1 - x_1^2(t)]x_2(t) + u(t)$$

is to be controlled to minimize the performance measure

$$J = \int_0^1 \tfrac{1}{2}[2x_1^2(t) + x_2^2(t) + u^2(t)]\, dt.$$

The initial and final state values are specified.

(a) Determine the costate equations for the system.

(b) Determine the control that minimizes the Hamiltonian for:

(i) $u(t)$ not bounded.

5-3. The system given in Problem 5-2 is to be transferred from the origin to the plane

$$15x_1(t) + 20x_2(t) + 12t = 60$$

while the performance measure

$$J = \tfrac{1}{2}\int_0^{t_f} u^2(t)\, dt$$

is minimized. The final time t_f is free.

(a) Determine the costate equations for the system.

(b) Find the control that minimizes $\mathscr{H}$ for

(i) $u(t)$ not bounded.

5-6. A first-order system is described by the state equation

$$\dot{x}(t) = x(t) + u(t).$$

(a) Find the unconstrained control, in closed-loop form, which minimizes the functional

$$J = \int_0^T [1.5x^2(t) + 0.5u^2(t)]\, dt.$$

T is fixed, and $x(t_f)$ is free.

(b) Show that for $T \longrightarrow \infty$ the optimal control law is of the form

$$u^*(t) = Fx^*(t),$$

where F is a constant. Find F.

5-11. (a) Determine the optimal control law for the system

$$\dot{x}(t) = -x(t) + u(t)$$

to be transferred to the origin from an arbitrary initial state. The performance measure is

$$J = \int_0^1 \tfrac{1}{2}[3x^2(t) + u^2(t)]\, dt.$$

The admissible controls are not bounded.

(b) Determine the optimal control law for the system and performance measure in part (a) with $x(1)$ free.

5-12. (a) Find the control that transfers the system given in Example 5.1-1 from $\mathbf{x}(0) = \mathbf{0}$ to the line $x_1(t) + 5x_2(t) = 15$ and minimizes

$$J = \tfrac{1}{2}[x_1(2) - 5]^2 + \tfrac{1}{2}[x_2(2) - 2]^2 + \tfrac{1}{2}\int_0^2 u^2(t)\, dt.$$

(b) Determine the cost of control

$$\tfrac{1}{2}\int_0^2 u^{*2}(t)\, dt$$

for part (a) above, and for parts (a) and (b) of Example 5.1-1. Compare the control costs and discuss the comparison qualitatively.

Problem 2.8 Find the optimal control $u^*(t)$ of the plant

$$\begin{aligned} \dot{x}_1(t) &= x_2(t); \quad x_1(0) = 3, \quad x_1(2) = 0 \\ \dot{x}_2(t) &= -2x_1(t) + 5u(t); \quad x_2(0) = 5, \quad x_2(2) = 0 \end{aligned}$$

which minimizes the performance index

$$J = \frac{1}{2}\int_0^2 \left[x_1^2(t) + u^2(t)\right] dt.$$

Problem 2.9 A second order plant is described by

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -2x_1(t) - 3x_2(t) + 5u(t) \end{aligned}$$

and the cost function is

$$J = \int_0^\infty [x_1^2(t) + u^2(t)]dt.$$

Find the optimal control, when $x_1(0) = 3$ and $x_2(0) = 2$.

Problem 2.10 For a second order system

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -2x_1(t) + 3u(t) \end{aligned}$$

with performance index

$$J = 0.5x_1^2(\pi/2) + \int_0^{\pi/2} 0.5u^2(t)dt$$

and boundary conditions $\mathbf{x}(0) = [0 \;\; 1]'$ and $\mathbf{x}(t_f)$ is free, find the optimal control.

Problem 2.11 Find the optimal control for the plant

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -2x_1(t) + 3u(t)$$

with performance criterion

$$J = \frac{1}{2}F_{11}\left[x_1(t_f) - 4\right]^2 + \frac{1}{2}F_{22}\left[x_2(t_f) - 2\right]^2 + \frac{1}{2}\int_0^{t_f}\left[x_1^2(t) + 2x_2^2(t) + 4u^2(t)\right]dt$$

and initial conditions as $\mathbf{x}(0) = [1 \;\; 2]'$. The additional conditions are given below.

1. Fixed-final conditions $F_{11} = 0, F_{22} = 0, t_f = 2, \mathbf{x}(2) = [4 \;\; 6]'$.
2. Free-final time conditions $F_{11} = 3, F_{22} = 5, \mathbf{x}(t_f) = [4 \;\; 6]'$ and t_f is free.
3. Free-final state conditions, $F_{11} = F_{22} = 0, x_1(2)$ is free and $x_2(2) = 6$.
4. Free-final time and free-final state conditions, $F_{11} = 3, F_{22} = 5$ and the final state to have $x_1(t_f) = 4$ and $x_2(t_f)$ to lie on $\theta(t) = -5t + 15$.

Problem 3.1 A first order system is given by

$$\dot{x}(t) = x(t) + u(t).$$

(a) Find the unconstrained optimal control law which minimizes the performance index

$$J = \int_0^{t_f}[2x^2(t) + 0.25u^2(t)]dt,$$

such that the final time t_f is fixed and the final state $x(t_f)$ is free.
(b) Find the optimal control law as $t_f \to \infty$.

Problem 3.5 Find the closed-loop, unconstrained, optimal control for the system

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -2x_1(t) - 3x_2(t) + u(t)$$

and the performance index

$$J = \int_0^\infty [x_1^2(t) + x_2^2(t) + u^2(t)]dt.$$

Problem 3.8 Given a third order plant,

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = x_3(t)$$
$$\dot{x}_3(t) = -5x_1(t) + -7x_2(t) - 10x_3(t) + 4u(t)$$

and the performance index

$$J = \int_0^\infty [q_{11}x_1^2(t) + q_{22}x_2^2(t) + q_{33}x_3^2(t) + ru^2(t)]dt,$$

for

1. $q_{11} = q_{22} = q_{33} = 1, r = 1,$
2. $q_{11} = 10, q_{22} = 1, q_{33} = 1, r = 1,$ and
3. $q_{11} = q_{22} = q_{33} = 1, r = 10,$

find the positive definite solution for Riccati coefficient matrix $\bar{\mathbf{P}}$, optimal feedback gain matrix $\bar{\mathbf{K}}$ and the eigenvalues of the closed-loop system matrix $\mathbf{A} - \mathbf{B}\bar{\mathbf{K}}$.

Problem 3.9 Determine the optimal feedback coefficients and the optimal control law for the multi-input, multi-output (MIMO) system

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \mathbf{u}(t)$$

and the cost function

$$J = \int_0^\infty [2x_1^2(t) + 4x_2^2(t) + 0.5u_1^2(t) + 0.25u_2^2(t)]dt.$$

Problem #

A second order plant

$$\dot{x}_1(t) = x_2(t),$$
$$\dot{x}_2(t) = -2x_1(t) - 3x_2(t) + u(t)$$
$$\mathbf{y}(t) = \mathbf{x}(t)$$

is to be controlled to minimize the performance index

$$J = \int_{t_0}^{t_f} \left[[2t - x_1(t)]^2 + 0.02u^2(t) \right] dt \qquad (4.1.49)$$

where, t_f is specified and $\mathbf{x}(t_f)$ is free. Find the optimal control in order that the state $x_1(t)$ *track* a ramp function $z_1(t) = 2t$ and without much expenditure of control energy. Plot all the variables (Riccati coefficients, optimal states and control) for initial conditions $\mathbf{x}(0) = [-1 \;\; 0]'$.

Problem 4.1 A second order plant

$$\dot{x}_1(t) = x_2(t),$$
$$\dot{x}_2(t) = -2x_1(t) - 3x_2(t) + u(t)$$
$$\mathbf{y}(t) = \mathbf{x}(t)$$

is to be controlled to minimize a performance index and to keep the state $x_1(t)$ close to a ramp function $2t$. The final time t_f is specified, the final state $\mathbf{x}(t_f)$ is free and the admissible controls and states are unbounded. Formulate the performance index, obtain the feedback control law and plot all the time histories of Riccati coefficients, optimal states and control.

Problem 4.2 A second order plant

$$\begin{aligned}\dot{x}_1(t) &= x_2(t),\\ \dot{x}_2(t) &= -2x_1(t) - 4x_2(t) + 0.5u(t)\\ \mathbf{y}(t) &= \mathbf{x}(t)\end{aligned}$$

is to be controlled to minimize the performance index

$$J = \int_{t_0}^{t_f} \left[4x_1^2(t) + 6x_2^2(t) + 0.02u^2(t)\right] dt.$$

The final time t_f is specified, the final state $\mathbf{x}(t_f)$ is fixed and the admissible controls and states are unbounded. Obtain the feedback control law and plot all the time histories of inverse Riccati coefficients, optimal states and control.

Problem 4.4 Using the frequency-domain results, determine the optimal feedback coefficients and the closed-loop optimal control for the multi-input, multi-output system

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{u}(t)$$

and the cost function

$$J = \int_0^{\infty} [4x_1^2(t) + 4x_2^2(t) + 0.5u_1^2(t) + u_2^2(t)]dt.$$