

**JAM 2006**

**GEOPHYSICS TEST PAPER**

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**IMPORTANT NOTE FOR CANDIDATES**

Select any **Two** Sections. Attempt **ALL** objective and subjective questions of the same **Two** Sections. Questions 1- 45 (objective questions) carry **three** marks each and questions 46 - 66 (subjective questions) carry **fifteen** marks each.

Write the answers to the objective questions in the Answer Table for Objective Questions provided on page 17 only)

**GEOLOGY SECTION**

1. Plutonic equivalent of Trachyte is
  - (A) diorite
  - (B) gabbro
  - (C) granite
  - (D) syenite
2. The river meanders at the mature stage with gentle gradient. The formation of pointbar will be on:
  - (A) outer zone of the bend
  - (B) inner zone of the bend
  - (C) straight channel segment
  - (D) steep bank of the channel.
3. Match the features in **Group 1** with the agents responsible for these features from **Group 2**

**Group 1**

- P. Arete  
Q. Backswamp  
R. Yardangs  
S. Stalactites and stalagmites

**Group 2**

1. River  
2. Ground Water  
3. Glacier  
4. Wind

Choose the correct answer from the following:

- (A) P-4, Q-3, R-1, S-2
  - (B) P-2, Q-1, R-3, S-4
  - (C) P-3, Q-1, R-4, S-2
  - (D) P-1, Q-2, R-4, S-3
4. The area bounded by two fault planes dipping away from each other with hanging walls going downward is called as:
    - (A) dome
    - (B) grabben
    - (C) horst
    - (D) klippe

5. Match the characteristics in **Group 1** with the structures in **Group 2**

**Group 1**

- P. axial plane is horizontal
- Q. hinges are sharp and angular
- R. limbs are parallel
- S. thickness of bed remains constant

**Group 2**

- 1. Isoclinal fold
- 2. Parallel fold
- 3. Recumbant fold
- 4. Chevron fold

Choose the correct answer from the following:

- (A) P-2, Q-4, R-2, S-1
- (B) P-1, Q-3, R-2, S-4
- (C) P-4, Q-2, R-1, S-3
- (D) P-3, Q-4, R-1, S-2

6. Which of the following rock indicates initiation of metamorphism

- (A) phyllite
- (B) schist
- (C) shale
- (D) slate

7. Barrovian metamorphism of pelitic rocks is characterized by the first appearance of index minerals in a particular sequence. Which one of the following is the correct sequence.

- (A) chlorite-garnet-biotite-kyanite-staurolite-sillimanite
- (B) garnet-biotite-chlorite-staurolite-sillimanite-kyanite
- (C) chlorite-biotite-garnet-staurolite-kyanite-sillimanite
- (D) biotite-chlorite-garnet-kyanite-staurolite-sillimanite

8. A crystal has three crystallographic axes of 2 fold symmetry and mirror plane perpendicular to each of these crystallographic axes. The Herman-Mauguin notation for crystal would be

- (A)  $2/m\ 2/m\ 2/m$
- (B)  $2m$
- (C)  $2mm$
- (D)  $23$

9. Indicate the correct order in terms of increasing Si : O ratio.

- (A) phlogopite-beryl-plagioclase-epidote
- (B) epidote-beryl-phlogopite-plagioclase
- (C) beryl-phlogopite-plagioclase-epidote
- (D) plagioclase-phlogopite-epidote-beryl

10. Sandstones and purple shales of Muree Series of Potwar region, equivalent to Dagshai and Kasauli beds of northwest Himalaya belongs to

- (A) Upper Eocene
- (B) Lower Eocene
- (C) Middle Miocene
- (D) Lower Miocene

11. A radiogenic isotope has half-life of 1 hour and we have 10000 atoms of that particular isotope in a particular system at a particular time. How much atoms of that isotope will be there after 6 hours?
- (A) 78  
(B) 156  
(C) 313  
(D) 625
12. Find the odd man out from the following
- (A) stockwork  
(B) ladder vein  
(C) saddle reef  
(D) banding
13. Sulfide chimneys are observed at
- (A) vents of seafloor hotsprings around ridges  
(B) inland hotspring vents in volcanic terrains  
(C) sulfide mineral coatings on the chimneys of smelters  
(D) mouths of explosive volcanoes
14. Within the mantle sudden density change produce seismic-wave discontinuities due to polymorphic transition or compositional change or a combination of both occur at a depth of
- (A) 470 kms  
(B) 570 kms  
(C) 670 kms  
(D) 760 kms
15. The estimated thickness of the moon's lithosphere is about
- (A) 35 km  
(B) 65 km  
(C) 100 km  
(D) 1000 km

**PHYSICS SECTION**

16. In case of an inelastic collision which one of the following is true
- (A) Total energy is not conserved  
(B) Momentum is not conserved  
(C) Kinetic energy is conserved  
(D) Kinetic energy is not conserved
17. The root mean square speed of an ideal gas, made up of molecules of molecular weight 0.0831 kg/mol, at temperature 300<sup>0</sup> K is (Take universal gas constant  $R= 8.31$  J/mol K)
- (A) 100 m/s  
(B) 200 m/s  
(C) 300 m/s  
(D) 400 m/s

18. The temperature differences between hot ( $T_H$ ) and cold ( $T_C$ ) reservoirs of two Carnot engines A and B are the same. If the ratio of the respective efficiencies,  $\frac{\eta^A}{\eta^B}$ , is equal to  $\frac{1}{2}$  then the ratio of the hot reservoir temperatures  $\frac{T_H^A}{T_H^B}$  is
- (A) 0.25  
 (B) 0.5  
 (C) 1.0  
 (D) 2.0
19. Which one of the following phenomenon cannot be described by the particle nature of electromagnetic radiations.
- (A) Blackbody radiations  
 (B) Compton scattering  
 (C) Photoelectric effect  
 (D) X-ray diffraction
20. If a semiconductor is doped with donor atoms then the impurity levels created in the semiconductor are close to the
- (A) bottom of the conduction band  
 (B) top of the valence band  
 (C) bottom of the valence band  
 (D) top of the conduction band
21. Binding energy per nucleon for the nuclei  ${}^4\text{He}$ ,  ${}^{56}\text{Fe}$ ,  ${}^{197}\text{Au}$  and  ${}^{235}\text{U}$  are given by  $B_1$ ,  $B_2$ ,  $B_3$  and  $B_4$ , respectively. These binding energies satisfy the order
- (A)  $B_1 < B_2 < B_3 < B_4$   
 (B)  $B_1 > B_2 > B_3 > B_4$   
 (C)  $B_2 < B_3 < B_4 < B_1$   
 (D)  $B_2 > B_3 > B_4 > B_1$
22. When a thin transparent sheet is introduced along the path of one of the slits in Young's double slit experiment, then the fringe width
- (A) decreases  
 (B) increases  
 (C) does not change  
 (D) does not change but intensity becomes half

23. An infinite wire, lying along the  $z$ -axis, carries a current  $I$  in the positive  $z$  direction denoted by  $\hat{k}$ . The magnetic field at a point  $d \hat{i}$  is
- (A)  $\frac{\mu_0 I}{2\pi d} \hat{j}$
- (B)  $\frac{\mu_0 I}{2\pi d} \hat{i}$
- (C)  $-\frac{\mu_0 I}{2\pi d} \hat{j}$
- (D)  $-\frac{\mu_0 I}{2\pi d} \hat{i}$
24. The radius of curvature of curved surface of a plano-convex thin lens of glass (refractive index  $n = 1.5$ ) of focal length  $0.4$  m is
- (A)  $0.1$  m
- (B)  $0.2$  m
- (C)  $0.4$  m
- (D)  $0.8$  m
25. The engine of a train, emitting the sound of frequency  $\nu_0$  approaches an observer with constant speed. If the observer measures the frequencies as  $\nu_1$  when it is approaching and  $\nu_2$  while it is going away, the relation between the frequencies is given by
- (A)  $\nu_1 = \nu_2 = \nu_0$
- (B)  $\nu_1 > \nu_0 > \nu_2$
- (C)  $\nu_1 < \nu_0 < \nu_2$
- (D)  $\nu_1 = \nu_2 \neq \nu_0$
26. In a dielectric sphere the polarization  $\vec{P}$  is given by  $\vec{P} = kr^3 \hat{r}$ . The corresponding bound volume charge density is equal to
- (A)  $-20k$
- (B)  $-10k$
- (C)  $10k$
- (D)  $20k$
27. An ideal fluid is flowing through a tube of cylindrical cross section with smoothly varying radius. The velocity of fluid particles at the point where tube's cross sectional area is  $1 \times 10^{-4} \text{ m}^2$  is given by  $0.01$  m/s. The velocity at a point where cross sectional area is  $2 \times 10^{-4} \text{ m}^2$  is given by
- (A)  $0.0025$  m/s
- (B)  $0.005$  m/s
- (C)  $0.02$  m/s
- (D)  $0.04$  m/s

28. The solution of Maxwell's equation for electric field in free space is given by  $E = E_0 \sin \omega(t - x/c)$ , where  $E_0$  is a constant,  $\omega$  is the angular frequency and  $c$  is the speed of light. The corresponding solution for the magnetic field  $B$  is
- (A)  $B = c E_0 \sin \omega(t - x/c)$   
 (B)  $B = \frac{E_0}{c} \sin \omega(t - x/c)$   
 (C)  $B = \frac{E_0}{c^2} \sin \omega(t - x/c)$   
 (D)  $B = \frac{E_0}{c^3} \sin \omega(t - x/c)$
29. The frequency of electron in  $n=1$  Bohr orbit is given by  $f_1$  revolutions/s. The frequency of electron in the  $n$ -th orbit for  $n > 1$  is
- (A)  $f_1/n$   
 (B)  $f_1/n^2$   
 (C)  $f_1/n^3$   
 (D)  $n f_1$
30. A signal of 1 mV is input to an amplifier circuit consisting of a transistor in common-emitter mode. What is the voltage gain if the collector current changes by 1 mA and the load resistance is equal to 1 k $\Omega$ ?
- (A) 10  
 (B)  $10^2$   
 (C)  $10^3$   
 (D)  $10^4$

### MATHEMATICS SECTION

31. Let  $\sum_{n \geq 1} a_n, a_n > 0$  be a convergent series. Now, consider the following statements:

**P:** The series  $\sum_{n \geq 1} \sqrt{a_n}$  is always convergent.

**Q:** The series  $\sum_{n \geq 1} \left( \frac{a_1 + a_2 + \dots + a_n}{n} \right)$  is always divergent.

Then,

- (A) both **P** and **Q** are true  
 (B) **P** is true and **Q** is false  
 (C) both **P** and **Q** are false  
 (D) **P** is false and **Q** is true

32. Let  $f : [0,1] \rightarrow [0,1]$  be defined by

$$f(x) = \begin{cases} \frac{1}{2} + \left(x - \frac{1}{2}\right)^2, & \text{if } x \text{ is rational} \\ \frac{1}{2}, & \text{if } x \text{ is irrational.} \end{cases}$$

Then,

- (A)  $f$  is continuous and differentiable only at  $x = \frac{1}{2}$
- (B)  $f$  is continuous only at  $x = \frac{1}{2}$  but not differentiable at  $x = \frac{1}{2}$
- (C)  $f$  is neither continuous nor differentiable at  $x = \frac{1}{2}$
- (D)  $f$  is continuous and differentiable for every  $x \in [0,1]$

33. The value of the integral  $\oint_C \frac{dz}{(z-i)^2(z+i)}$ , where  $C = \{z : |z-i|=1\}$ , is

- (A) 1
- (B)  $\pi$
- (C)  $\frac{\pi}{2}i$
- (D)  $\pi i$

34. The integral  $\int_0^1 \int_0^x \int_0^y f(x, y, z) dz dy dx$  is equal to

- (A)  $\int_0^1 \int_0^x \int_0^x f(x, y, z) dy dz dx$
- (B)  $\int_0^1 \int_0^x \int_0^z f(x, y, z) dy dz dx$
- (C)  $\int_0^1 \int_0^1 \int_0^x f(x, y, z) dy dz dx$
- (D)  $\int_0^1 \int_0^1 \int_0^z f(x, y, z) dy dz dx$



35. Consider the initial value problem (IVP):  $x y' - y = 0$ ,  $y(0)=0$ . Now, consider the following statements:

**P:** Picard's theorem is applicable to the above IVP.

**Q:** The above IVP has exactly one solution.

Then,

- (A) both **P** and **Q** are true
- (B) **P** is false but **Q** is true
- (C) both **P** and **Q** are false
- (D) **P** is true but **Q** is false

36. Let  $Q$  be the set of rational numbers in  $\mathbb{R}$ . Then

- (A)  $Q$  is closed in  $\mathbb{R}$
- (B)  $Q$  is open in  $\mathbb{R}$
- (C)  $Q$  is both open and closed in  $\mathbb{R}$
- (D)  $Q$  is neither open nor closed in  $\mathbb{R}$

37. The radius of convergence of the power series  $\sum_{n \geq 0} \frac{(n!)^2}{(2n)!} x^{2n}$  is

- (A)  $\frac{1}{2}$
- (B)  $\frac{1}{\sqrt{2}}$
- (C)  $\sqrt{2}$
- (D) 2

38. Consider the differential equation  $y'' + 6y' + 25y = 0$  with initial condition  $y(0)=0$ . Then, the general solution of the IVP is

- (A)  $e^{-3x}(A \cos 4x + B \sin 4x)$
- (B)  $B e^{-3x} \sin 4x$
- (C)  $A e^{-4x} \sin 3x$
- (D)  $e^{-4x}(A \cos 3x + B \sin 3x)$

39. Let  $\vec{F}(x, y, z) = x^2 y \hat{i} + y \hat{j} + z^2 \hat{k}$ . If  $\vec{p} = \text{curl } \vec{F}$  and  $q = \text{div } \vec{F}$ , then  $(\vec{p}, q)$  is

- (A)  $(-x^2 \hat{k}, 1 + 2xy + 2z)$
- (B)  $(2xy \hat{i} + \hat{j} + 2z \hat{k}, 1 + 2xy + 2z)$
- (C)  $(-x^2 \hat{k}, x^2 y + y + z^2)$
- (D)  $(2xy \hat{i} + \hat{j} + 2z \hat{k}, x^2 y + y + z^2)$

40. Let  $V = \{ (x, y, z, w) : x+y+z-3w=0, x-y+z-w=0, x-7y+z+5w=0 \}$  be a vector subspace of  $\mathfrak{R}^4$ . Then  $\dim(V)$  is

- (A) 1
- (B) 2
- (C) 3
- (D) 4

41. Let  $T : \mathfrak{R}^3 \rightarrow \mathfrak{R}^3$  be a linear transformation defined by  $T(x, y, z) = (x+y+z, y+z, z)$ . Then  $T^n(x, y, z)$ , for  $n \geq 1$ , is

- (A)  $\left( x+ny+\frac{n^2+n}{2}z, y+nz, z \right)$
- (B)  $\left( x+ny+(n^2-n+1)z, y+nz, z \right)$
- (C)  $\left( x+ny+\frac{n^2+5n-2}{4}z, y+nz, z \right)$
- (D)  $\left( x+ny+\frac{3n^2-n+2}{4}z, y+nz, z \right)$

42. Suppose that the moment generating function of a random variable  $X$  is  $\frac{1}{2}e^{-3t} + \frac{1}{4}e^{-2t} + \frac{1}{4}e^{2t}$ . Then  $\text{Var}(X)$  is

- (A)  $\frac{3}{2}$
- (B)  $\frac{17}{4}$
- (C)  $\frac{13}{2}$
- (D)  $\frac{35}{4}$

43. Perform Newton's method to the equation  $x^3 - x - 2 = 0$  starting with  $x_0 = 1$ . In this operation, the value of  $x_2$  (the second iterate) is

- (A)  $\frac{3}{4}$
- (B)  $\frac{1}{2}$
- (C)  $\frac{18}{11}$
- (D) 2

44. The distribution function  $F$  of a random variable  $X$  is

$$F(x) = \begin{cases} 0, & \text{if } x < -1 \\ 1/8, & \text{if } -1 \leq x < 0 \\ 1/4, & \text{if } 0 \leq x < 1 \\ 1/2, & \text{if } 1 \leq x < 2 \\ 1, & \text{if } x \geq 2. \end{cases}$$

If  $\alpha = P\left(-\frac{1}{2} < X \leq 1\right)$  and  $\beta = P(0 \leq X < 2)$ , then  $(\alpha, \beta)$  is

- (A)  $\left(\frac{3}{8}, \frac{3}{8}\right)$
- (B)  $\left(\frac{1}{8}, \frac{3}{8}\right)$
- (C)  $\left(\frac{3}{8}, \frac{7}{8}\right)$
- (D)  $\left(\frac{1}{8}, \frac{1}{4}\right)$

45. Let  $X_1, X_2, X_3, \dots, X_n$  be a random sample from a normal population  $N(\mu, \sigma^2)$ , where  $\mu$  and  $\sigma^2$  are unknowns. Suppose that  $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ , where  $\bar{X}$  is the sample mean. It is known that  $cS^2$  follows a  $\chi^2$ -distribution with  $(n-1)$  degrees of freedom. Then  $c$  is equal to

- (A)  $\frac{n}{\sigma}$
- (B)  $\frac{n}{\sigma^2}$
- (C)  $\frac{n-1}{\sigma}$
- (D)  $\frac{n-1}{\sigma^2}$

### GEOLOGY SECTION

46. (a) What is the relationship between an earthquake focus and the corresponding epicenter? (6)  
 (b) What are the three kinds of Plate margins and associated magmatism? (9)

47. (a) What is dip slip fault? In an area a bed is dipping towards west at  $42^\circ$ . The area had been affected by fault dipping toward east at  $45^\circ$ . With the help of neat diagrams show the relative movements of the blocks resulting in repetition of bed and and omission of bed (9)

- (b) How you define monocline? A N-S trending bed is exposed on an easterly sloping ground with the bed-dipping west. Find the thickness of the bed, if the slope of the ground is  $15^\circ$  E; width of the bed measured perpendicular to strike is 100 m; dip of the bed is  $30^\circ$ W. (6)
48. (a) Why startovolcano like Mount Fuji in Japan has steep sides and shield volcano like Mauna Loa in Hawaii have gentle surface slopes? (6)  
 (b) How you define conformable and unconformable sequence? What geological events are indicated by angular unconformity? (9)
49. (a) Where do back-arc basins form and what is the necessary conditions for the formation of back-arc basins? How is the nature of magmatism different from that of a forearc? (9)  
 (b) Compare Airy's and Pratt's hypothesis on isostasy with the help of a neat diagram. (6)
50. (a) Distinguish between "perthitic" and "rapakivi" texture with the help of neat sketch. (6)  
 (b) Give the idealized Bouma sequence. Where do you find such a sequence of deposition of sediments? (9)
51. (a) Mention the broad tectonic regime and mode of occurrence of porphyry-copper deposits. (6)  
 (b) Mention 3 locality of each of occurrences of Iron, Manganese and Copper deposits in India. (9)
52. (a) A grain of undeformed quartz is in contact with an untwined plagioclase, both showing first order gray interference color. How do you distinguish the two? (6)  
 (b) What is an optical indicatrix? Draw a positive biaxial indicatrix indicating the optic axes, the optic axial angle and circular sections. (9)

### PHYSICS SECTION

53. An ideal diatomic gas at pressure  $p_i$  and volume  $V_i$  doubles its volume adiabatically. Find  
 (a) the final pressure and (6)  
 (b) the work done by the gas. (9)
54. A charge of magnitude  $9.8 \times 10^{-10}$  C and mass  $2.0 \times 10^{-6}$  kg is suspended through a silk thread along the line passing through the center and parallel to the length of two parallel plates with a spacing of 0.1 m. The plates are connected to a voltage source of 2000 V. (Take  $g = 9.8 \text{ ms}^{-2}$ ). Find  
 (a) the electric field experienced by the charge and (6)  
 (b) the angle that the thread makes with the vertical when charge is in equilibrium. (9)

55. A cylinder of 1 kg mass and 0.02 m diameter left at the top of an inclined plane of height 1 m rolls down without slipping. (Take  $g = 9.8 \text{ ms}^{-2}$ )
- What is the kinetic energy of the cylinder when it reaches at the bottom of inclined plane? (6)
  - Find the velocity of center of mass of cylinder on reaching the bottom of inclined plane. (9)
56. Two waves described by  $y_1 = A \sin(\omega t + kx)$  and  $y_2 = A \sin(\omega t - kx)$  are traveling along a string. Let  $A = 0.001 \text{ m}$ ,  $k = 3.142 \text{ m}^{-1}$  and  $\omega = 157.1 \text{ s}^{-1}$  (Take  $\pi = 3.142$ )
- Find the magnitude and direction of velocity of these waves. (6)
  - What shall be the amplitude of resultant wave on the string at  $x = 0.5 \text{ m}$ . (9)
57. Consider a monatomic FCC solid with lattice constant  $\sqrt{3} \text{ \AA}$ .
- Find the interplanar spacing of a set of parallel (111) planes. (6)
  - For what incident angle  $\theta$  the first order Bragg peak would be observed if a monochromatic x-ray of wavelength  $1 \text{ \AA}$  is incident on these planes? (9)
58. Consider an LR circuit with an inductor  $L$ , a resistor  $R$ , a battery of emf  $E$  and a switch  $S$ , all connected in series.
- Find an expression for current  $I$  in the circuit as a function of time after the switch  $S$  is closed. (9)
  - What is value of  $I$  after a time that equals the time constant of this circuit? (6)
59. Take radius of hydrogen atom  $H$  to be  $5.3 \times 10^{-11} \text{ m}$ . (Take  $\hbar = 1.054 \times 10^{-34} \text{ J s}$  and  $m_e = 9.1 \times 10^{-31} \text{ kg}$ ) Assuming momentum of electron to be same as order of uncertainty in momentum,
- Find the order of kinetic energy that an electron in the hydrogen atom is expected to have based on the uncertainty principle. (9)
  - If de Broglie wavelength of electron matches with the circumference of orbit, what is the velocity of electron? (6)

### Mathematics Section

60. (a) Suppose that  $f : [a, b] \rightarrow \mathbb{R}$ ,  $a > 0$  is continuous on the closed interval  $[a, b]$ , that  $f$  is differentiable on the open interval  $(a, b)$ , and that  $b f(a) = a f(b)$ . Then prove that there exists  $c \in (a, b)$  such that  $f(c) = c f'(c)$ . (6)
- (b) Let  $f : [0, 2] \rightarrow \mathbb{R}$  be defined by  $f(x) = \frac{x}{2} + (x-1)^{\frac{2}{3}}$ . Compute the absolute maximum and minimum value of  $f$  on  $[0, 2]$ . (9)

61. (a) Let  $f : [0, 1] \rightarrow \mathfrak{R}$  be continuous with  $\int_0^x f(t) dt = \int_x^1 f(t) dt$  for all  $x \in [0, 1]$ . Does the above condition imply that  $f(x) \equiv 0$  on  $[0, 1]$ ? Explain. (6)
- (b) Let  $f : [0, 1] \rightarrow \mathfrak{R}$  be defined by  $f(x) = x^3$ . Find the area of the surface generated by revolving the curve  $y = f(x)$  about the  $x$ -axis. (9)
62. (a) Let  $f(x) = 1 + 3x^2 + 5x^4 + 7x^6 + \dots$ , for  $|x| < 1$ , be a power series. Determine  $f\left(\frac{1}{2}\right)$ . (6)
- (b) Let  $V$  be a vector subspace of  $\mathfrak{R}^4$  spanned by the vectors  $(1, 1, 1, -1)$  and  $(1, -1, 0, 1)$ . Let  $W$  be another vector subspace of  $\mathfrak{R}^4$  spanned by the vectors  $(1, 1, -1, 1)$  and  $(1, 3, 4, -5)$ . Determine a basis for  $V \cap W$ . (9)
63. (a) Consider the system of linear equations
 
$$\begin{aligned} x + y + z &= 3 \\ x + 2y + 3z &= a \\ x + 3y + bz &= 5. \end{aligned}$$
 Determine the values for  $a$  and  $b$  for which the above system has a unique solution, infinite number of solutions, and no solution. (9)
- (b) Solve:  $(4x^2y + 5x^3y^2) dx + (2x^3 + 3x^4y) dy = 0$ . (6)
64. (a) Let  $C$  be the boundary of the triangle with vertices  $(0, 1, 0)$ ,  $(1, 0, 0)$  and  $(2, 1, 0)$ . If  $\vec{F}(x, y, z) = -y\hat{i} + y^2z\hat{j} + zx\hat{k}$ , then use Stoke's theorem to evaluate  $\int_C \vec{F} \cdot d\vec{r}$  when  $C$  is traversed counter-clockwise when viewed from above. (9)
- (b) Let  $u(x, y) = x^3 - 3xy^2 + x + 3$  be the real part of an analytic function  $f(x, y)$  on the entire complex plane. Determine the harmonic conjugate of  $u(x, y)$ . (6)
65. (a) Let  $X_1, X_2, X_3, \dots, X_{20}$  be a random sample of size 20 from a normal population  $N(0, \sigma^2)$ . Find the best critical region of size  $\alpha = 0.05$  for testing  $H_0 : \sigma^2 = 1$  against  $H_1 : \sigma^2 = 2$ . (9)
- [Given:  $\chi_{20}^2(0.95) = 31.4$ ,  $\chi_{19}^2(0.95) = 30.1$ ,  $\chi_{20}^2(0.05) = 10.9$ , and  $\chi_{19}^2(0.05) = 10.1$ ]
- (b) Let  $X_1, X_2, X_3, \dots, X_n$  be a random sample of size  $n$  from a normal population  $N(\mu, 16)$ . Compute the minimum integral value of  $n$  such that  $P(\bar{X} - 2 < \mu < \bar{X} + 2) \geq 0.95$ , where  $\bar{X}$  is the sample mean. (6)
- [For  $Z \sim N(0, 1)$  and  $\Phi(z) = P(-\infty < Z < z)$ ,  $\Phi(1.645) = 0.95$  and  $\Phi(1.96) = 0.975$ ]

66. (a) Determine the value of  $c$  so that

$$f(x, y) = \begin{cases} c(x^2 - y^2), & \text{for } (x, y) \in D \\ 0, & \text{otherwise,} \end{cases}$$

where  $D$  is the triangle with vertices  $(0,0)$ ,  $(2,0)$  and  $(2,2)$ , is the joint probability density function of the random variables  $X$  and  $Y$ . (6)

- (b) The table below gives the values of  $f(x)$  for  $1 \leq x \leq 9$ .

$x$	1	3	5	7	9
$f(x)$	1	0	1	0	1

Compute the forward difference table and determine  $f(2)$  up to four decimal places. (9)