JAM 2015: General Instructions during Examination

1. Total duration of the JAM 2015 examination is **180** minutes.
2. The clock will be set at the server. The countdown timer at the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You need not terminate the examination or submit your paper.
3. Any useful data required for your paper can be viewed by clicking on the **Useful Data** button that appears on the screen.
4. Use the scribble pad provided to you for any rough work. Submit the scribble pad at the end of the examination.
5. You are allowed to use only your own **non-programmable calculator**.
6. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:

   - 1 You have not visited the question yet.
   - 3 You have not answered the question.
   - 5 You have answered the question.
   - 7 You have NOT answered the question, but have marked the question for review.
   - 9 You have answered the question, but marked it for review.

7. The **Marked for Review** status for a question simply indicates that you would like to look at that question again. *If a question is 'answered, but marked for review', then the answer will be considered for evaluation unless the status is modified by the candidate.*

Navigating to a Question :

8. To answer a question, do the following:
   a. Click on the question number in the Question Palette to go to that question directly.
   b. Select the answer for a multiple choice type question and for the multiple select type question. Use the virtual numeric keypad to enter the answer for a numerical type question.
   c. Click on **Save & Next** to save your answer for the current question and then go to the next question.
   d. Click on **Mark for Review & Next** to save and to mark for review your answer for the current question, and then go to the next question.

Caution: Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on a question number without saving the answer to the previous question.

9. You can view all the questions by clicking on the **Question Paper** button. This feature is provided, so that if you want you can just see the entire question paper at a glance.

Answering a Question :

10. Procedure for answering a multiple choice question (MCQ):
   a. Choose the answer by selecting only one out of the 4 choices (A,B,C,D) given below the question and click on the bubble placed before the selected choice.

b. To deselect your chosen answer, click on the bubble of the selected choice again or click on the **Clear Response** button.

c. To change your chosen answer, click on the bubble of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

11. Procedure for answering a multiple select question (MSQ):

a. Choose the answer by selecting one or more than one out of the 4 choices (A,B,C,D) given below the question and click on the checkbox(es) placed before each of the selected choice (s).

b. To deselect one or more of your selected choice(s), click on the checkbox(es) of the choice(s) again. To deselect all the selected choices, click on the **Clear Response** button.

c. To change a particular selected choice, deselect this choice that you want to change and click on the checkbox of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

12. Procedure for answering a numerical answer type (NAT) question:

a. To enter a number as your answer, use the virtual numerical keypad.

b. A fraction (e.g. -0.3 or -.3) can be entered as an answer with or without '0' before the decimal point. As many as four decimal points, e.g. 12.5435 or 0.003 or -932.6711 or 12.82 can be entered.

c. To clear your answer, click on the **Clear Response** button.

d. To save your answer, you MUST click on the **Save & Next** button.

13. To mark a question for review, click on the **Mark for Review & Next** button. *If an answer is selected (for MCQ and MSQ types) or entered (for NAT) for a question that is* ***Marked for Review****, that answer will be considered in the evaluation unless the status is modified by the candidate.*

14. To change your answer to a question that has already been answered, first select that question and then follow the procedure for answering that type of question as described above.

15. Note that ONLY those questions for which answers are ***saved*** or ***marked for review after answering*** will be considered for evaluation.

Choosing a Section :

16. Sections in this question paper are displayed on the top bar of the screen. All sections are compulsory.

17. Questions in a section can be viewed by clicking on the name of that section. The section you are currently viewing will be highlighted.

18. To select another section, simply click the name of the section on the top bar. You can shuffle between different sections any number of times.

19. When you select a section, you will only be able to see questions in this Section, and you can answer questions in the Section.

20. After clicking the **Save & Next** button for the last question in a section, you will automatically be taken to the first question of the next section in sequence.

21. You can move the mouse cursor over the name of a section to view the answering status for that section.

JAM 2015 Examination

MA: Mathematics

Duration: **180 minutes** *Maximum Marks*: **100**

Read the following instructions carefully.

1. To login, enter your Registration Number and Password provided to you. Kindly go through the various coloured symbols used in the test and understand their meaning before you start the examination.
2. Once you login and after the start of the examination, you can view all the questions in the question paper, by clicking on the **Question Paper** button in the screen.
3. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into three **sections, A, B** and **C**. All sections are compulsory. Questions in each section are of different types.
4. **Section – A** contains **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only one choice is the correct answer. This section has 30 Questions and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.
5. **Section – B** contains **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct choices only and no wrong choices. This section has 10 Questions and carry 2 marks each with a total of 20 marks.
6. **Section – C** contains **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual numerical keypad on the monitor. No choices will be shown for these type of questions. This section has 20 Questions and carry a total of 30 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.20 carry 2 marks each.
7. Depending upon the JAM test paper, there may be useful common data that may be required for answering the questions. If the paper has such useful data, the same can be viewed by clicking on the **Useful Data** button that appears at the top, right hand side of the screen.
8. The computer allotted to you at the examination centre runs specialized software that permits only one choice to be selected as answer for multiple choice questions using a mouse, one or more than one choices to be selected as answer for multiple select questions using a mouse and to enter a suitable number for the numerical answer type questions using the virtual numeric keypad and mouse.
9. Your answers shall be updated and saved on a server periodically and also at the end of the examination. The examination will **stop automatically** at the end of **180 minutes**.
10. Multiple choice questions (Section-A) will have four choices against A, B, C, D, out of which only **ONE** choice is the correct answer. The candidate has to choose the correct answer by clicking on the bubble (○) placed before the choice.
11. Multiple select questions (Section-B) will also have four choices against A, B, C, D, out of which **ONE OR MORE THAN ONE** choice(s) is /are the correct answer. The candidate has to choose the correct answer by clicking on the checkbox (□) placed before the choices for each of the selected choice(s).
12. For numerical answer type questions (Section-C), each question will have a numerical answer and there will not be any choices**. For these questions, the answer should be entered** by using the mouse and the virtual numerical keypad that appears on the monitor.
13. In all questions, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

14. **Non-programmable calculators** are allowed but sharing of calculators is not allowed.

15. Mobile phones, electronic gadgets other than calculators, charts, graph sheets, and mathematical tables are **NOT** allowed in the examination hall.

16. You can use the scribble pad provided to you at the examination centre for all your rough work. The scribble pad has to be returned at the end of the examination.

Declaration by the candidate:

"I have read and understood all the above instructions. I have also read and understood clearly the instructions given on the admit card and shall follow the same. I also understand that in case I am found to violate any of these instructions, my candidature is liable to be cancelled. I also confirm that at the start of the examination all the computer hardware allotted to me are in proper working condition".

Notation

$\mathbb{N}$	- The set of natural numbers = $\{1,2,3,\ldots\}$
$\mathbb{Z}$	- The set of integers
$\mathbb{Q}$	- The set of rational numbers
$\mathbb{R}$	- The set of real numbers
$\mathbb{C}$	- The set of complex numbers
S_n	- The group of permutations of n distinct symbols
$\mathbb{Z}_n$	- The group of integers modulo n
$M_n(\mathbb{R})$	- The vector space of $n \times n$ real matrices
$\hat{\imath}, \hat{\jmath}, \hat{k}$	- Standard mutually orthogonal unit vectors
i	- Imaginary number $\sqrt{-1}$
$\bar{a}$	- Complex conjugate of a
$\bar{A}$	- Complex conjugate of matrix A
A^T	- Transpose of matrix A
$\emptyset$	- Empty set
sup	- supremum
inf	- infimum
y'	- Derivative of y

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Suppose N is a normal subgroup of a group G. Which one of the following is true?

(A) If G is an infinite group then G/N is an infinite group
(B) If G is a nonabelian group then G/N is a nonabelian group
(C) If G is a cyclic group then G/N is an abelian group
(D) If G is an abelian group then G/N is a cyclic group

Q.2 Let $y(x) = u(x)\sin x + v(x)\cos x$ be a solution of the differential equation $y'' + y = \sec x$. Then $u(x)$ is

(A) $\ln|\cos x| + C$ (B) $-x + C$
(C) $x + C$ (D) $\ln|\sec x| + C$

Q.3 Let a, b, c, d be distinct non-zero real numbers with $a + b = c + d$. Then an eigenvalue of the matrix $\begin{bmatrix} a & b & 1 \\ c & d & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is

(A) $a + c$ (B) $a + b$ (C) $a - b$ (D) $b - d$

Q.4 Let S be a nonempty subset of $\mathbb{R}$. If S is a finite union of disjoint bounded intervals, then which one of the following is true?

(A) If S is not compact, then $\sup S \notin S$ and $\inf S \notin S$
(B) Even if $\sup S \in S$ and $\inf S \in S$, S need not be compact
(C) If $\sup S \in S$ and $\inf S \in S$, then S is compact
(D) Even if S is compact, it is not necessary that $\sup S \in S$ and $\inf S \in S$

Q.5 Let $\{x_n\}$ be a convergent sequence of real numbers. If $x_1 > \pi + \sqrt{2}$ and $x_{n+1} = \pi + \sqrt{x_n - \pi}$ for $n \geq 1$, then which one of the following is the limit of this sequence?

(A) $\pi + 1$ (B) $\pi + \sqrt{2}$ (C) π (D) $\pi + \sqrt{\pi}$

Q.6 The volume of the portion of the solid cylinder $x^2 + y^2 \leq 2$ bounded above by the surface $z = x^2 + y^2$ and bounded below by the xy-plane is

(A) π (B) 2π (C) 3π (D) 4π

Q.7 Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0) = 0$. If for all $x \in \mathbb{R}$, $1 < f'(x) < 2$, then which one of the following statements is true on $(0, \infty)$?

(A) f is unbounded
(B) f is increasing and bounded
(C) f has at least one zero
(D) f is periodic

Q.8 If an integral curve of the differential equation $(y - x)\frac{dy}{dx} = 1$ passes through $(0, 0)$ and $(\alpha, 1)$, then α is equal to

(A) $2 - e^{-1}$
(B) $1 - e^{-1}$
(C) e^{-1}
(D) $1 + e$

Q.9 An integrating factor of the differential equation

$$\frac{dy}{dx} = \frac{2xy^2 + y}{x - 2y^3}$$

is

(A) $\frac{1}{y}$
(B) $\frac{1}{y^2}$
(C) y
(D) y^2

Q.10 Let A be a nonempty subset of $\mathbb{R}$. Let $I(A)$ denote the set of interior points of A. Then $I(A)$ can be

(A) empty
(B) singleton
(C) a finite set containing more than one element
(D) countable but not finite

Q. 11 – Q. 30 carry two marks each.

Q.11 Let S_3 be the group of permutations of three distinct symbols. The direct sum $S_3 \oplus S_3$ has an element of order

(A) 4
(B) 6
(C) 9
(D) 18

Q.12 The orthogonal trajectories of the family of curves $y = C_1 x^3$ are

(A) $2x^2 + 3y^2 = C_2$
(B) $3x^2 + y^2 = C_2$
(C) $3x^2 + 2y^2 = C_2$
(D) $x^2 + 3y^2 = C_2$

Q.13 Let G be a nonabelian group. Let $\alpha \in G$ have order 4 and let $\beta \in G$ have order 3. Then the order of the element $\alpha\beta$ in G

(A) is 6
(B) is 12
(C) is of the form $12k$ for $k \geq 2$
(D) need not be finite

Q.14 Let S be the bounded surface of the cylinder $x^2 + y^2 = 1$ cut by the planes $z = 0$ and $z = 1 + x$. Then the value of the surface integral $\iint_S 3z^2 d\sigma$ is equal to

(A) $\int_0^{2\pi} (1 + \cos\theta)^3 d\theta$

(B) $\int_0^{2\pi} \sin\theta \cos\theta \, (1 + \cos\theta)^2 d\theta$

(C) $\int_0^{2\pi} (1 + 2\cos\theta)^3 d\theta$

(D) $\int_0^{2\pi} \sin\theta \cos\theta \, (1 + 2\cos\theta)^2 d\theta$

Q.15 Suppose that the dependent variables z and w are functions of the independent variables x and y, defined by the equations $f(x, y, z, w) = 0$ and $g(x, y, z, w) = 0$, where $f_z g_w - f_w g_z = 1$. Which one of the following is correct?

(A) $z_x = f_w g_x - f_x g_w$

(B) $z_x = f_x g_w - f_w g_x$

(C) $z_x = f_z g_x - f_x g_z$

(D) $z_x = f_z g_w - f_z g_x$

Q.16 Let $A = \begin{bmatrix} 0 & 1 - i \\ -1 - i & i \end{bmatrix}$ and $B = A^T \overline{A}$. Then

(A) an eigenvalue of B is purely imaginary
(B) an eigenvalue of A is zero
(C) all eigenvalues of B are real
(D) A has a non-zero real eigenvalue

Q.17 The limit

$$\lim_{x \to 0+} \frac{1}{\sin^2 x} \int_{\frac{x}{2}}^{x} \sin^{-1} t \, dt$$

is equal to

(A) 0

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $\frac{3}{8}$

Q.18 Let $P_2(\mathbb{R})$ be the vector space of polynomials in x of degree at most 2 with real coefficients. Let $M_2(\mathbb{R})$ be the vector space of 2×2 real matrices. If a linear transformation $T: P_2(\mathbb{R}) \to M_2(\mathbb{R})$ is defined as

$$T(f) = \begin{bmatrix} f(0) - f(2) & 0 \\ 0 & f(1) \end{bmatrix}$$

then

(A) T is one-one but not onto
(B) T is onto but not one-one
(C) Range (T) = span $\left\{ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \right\}$
(D) Null (T) = span $\{x^2 - 2x, \; 1 - x\}$

Q.19

Let $B_1 = \{(1,2),\ (2,-1)\}$ and $B_2 = \{(1,0),\ (0,1)\}$ be ordered bases of $\mathbb{R}^2$. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $[T]_{B_1,B_2}$, the matrix of T with respect to B_1 and B_2, is $\begin{bmatrix} 4 & 3 \\ 2 & -4 \end{bmatrix}$, then $T(5,5)$ is equal to

(A) $(-9,\ 8)$ (B) $(9,\ 8)$ (C) $(-15,\ -2)$ (D) $(15,\ 2)$

Q.20

Let $S = \bigcap_{n=1}^{\infty}\left(\left[0,\ \frac{1}{2n+1}\right] \cup \left[\frac{1}{2n},\ 1\right]\right)$. Which one of the following statements is **FALSE**?

(A) There exist sequences $\{a_n\}$ and $\{b_n\}$ in $[0,1]$ such that $S = [0,1] \setminus \bigcup_{n=1}^{\infty}(a_n, b_n)$

(B) $[0,1]\backslash S$ is an open set

(C) If A is an infinite subset of S, then A has a limit point

(D) There exists an infinite subset of S having no limit points

Q.21 Let $f : \mathbb{R} \to \mathbb{R}$ be a strictly increasing continuous function. If $\{a_n\}$ is a sequence in $[0,1]$, then the sequence $\{f(a_n)\}$ is

(A) increasing (B) bounded

(C) convergent (D) not necessarily bounded

Q.22

Which one of the following statements is true for the series $\sum_{n=1}^{\infty}(-1)^n \frac{(2n)!}{n^{2n}}$?

(A) The series converges conditionally but not absolutely

(B) The series converges absolutely

(C) The sequence of partial sums of the series is bounded but not convergent

(D) The sequence of partial sums of the series is unbounded

Q.23

The sequence $\left\{\cos\left(\frac{1}{2}\tan^{-1}\left(-\frac{n}{2}\right)^n\right)\right\}$ is

(A) monotone and convergent

(B) monotone but not convergent

(C) convergent but not monotone

(D) neither monotone nor convergent

Q.24

If $y(t)$ is a solution of the differential equation $y'' + 4y = 2e^t$, then

$$\lim_{t\to\infty} e^{-t}\, y(t)$$

is equal to

(A) $\frac{2}{3}$ (B) $\frac{2}{5}$ (C) $\frac{2}{7}$ (D) $\frac{2}{9}$

Q.25

For what real values of x and y, does the integral $\int_x^y (6 - t - t^2)dt$ attain its maximum?

(A) $x = -3,\ y = 2$ (B) $x = 2,\ y = 3$
(C) $x = -2,\ y = 2$ (D) $x = -3,\ y = 4$

Q.26 The area of the planar region bounded by the curves $x = 6y^2 - 2$ and $x = 2y^2$ is

(A) $\frac{\sqrt{2}}{3}$ (B) $\frac{2\sqrt{2}}{3}$ (C) $\frac{4\sqrt{2}}{3}$ (D) $\sqrt{2}$

Q.27

For $n \geq 2$, let $f_n: \mathbb{R} \to \mathbb{R}$ be given by $f_n(x) = x^n \sin x$. Then at $x = 0$, f_n has a

(A) local maximum if n is even

(B) local maximum if n is odd

(C) local minimum if n is even

(D) local minimum if n is odd

Q.28

For $m, n \in \mathbb{N}$, define $f_{m,n}(x) = \begin{cases} x^m \sin\left(\frac{1}{x^n}\right), & x \neq 0 \\ 0 & x = 0 \end{cases}$

Then at $x = 0$, $f_{m,n}$ is

(A) differentiable for each pair m, n with $m > n$

(B) differentiable for each pair m, n with $m < n$

(C) not differentiable for each pair m, n with $m > n$

(D) not differentiable for each pair m, n with $m < n$

Q.29

Let G and H be nonempty subsets of $\mathbb{R}$, where G is connected and $G \cup H$ is not connected. Which one of the following statements is true for all such G and H ?

(A) If $G \cap H = \emptyset$, then H is connected

(B) If $G \cap H = \emptyset$, then H is not connected

(C) If $G \cap H \neq \emptyset$, then H is connected

(D) If $G \cap H \neq \emptyset$, then H is not connected

Q.30

Let $f : \{ (x,y) \in \mathbb{R}^2 : x > 0, \; y > 0 \} \to \mathbb{R}$ be given by

$$f(x,y) = x^{\frac{1}{3}} y^{\frac{-4}{3}} \tan^{-1}\left(\frac{y}{x}\right) + \frac{1}{\sqrt{x^2+y^2}}$$

Then the value of

$$g(x,y) = \frac{x f_x(x,y) + y f_y(x,y)}{f(x,y)}$$

(A) changes with x but not with y

(B) changes with y but not with x

(C) changes with x and also with y

(D) neither changes with x nor with y

SECTION - B
MULTIPLE SELECT QUESTIONS (MSQ)

Q. 1 – Q. 10 carry two marks each.

Q.1

Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \int_{-5}^{x} (t-1)^3 \, dt$.
In which of the following interval(s), f takes the value 1?

(A) $[-6, \; 0]$ (B) $[-2, \; 4]$ (C) $[2, \; 8]$ (D) $[6, \; 12]$

Q.2

Which of the following statements is (are) true?

(A) $\mathbb{Z}_2 \oplus \mathbb{Z}_3$ is isomorphic to $\mathbb{Z}_6$

(B) $\mathbb{Z}_3 \oplus \mathbb{Z}_3$ is isomorphic to $\mathbb{Z}_9$

(C) $\mathbb{Z}_4 \oplus \mathbb{Z}_6$ is isomorphic to $\mathbb{Z}_{24}$

(D) $\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_5$ is isomorphic to $\mathbb{Z}_{30}$

Q.3

Which of the following conditions implies (imply) the convergence of a sequence $\{x_n\}$ of real numbers?

(A) Given $\varepsilon > 0$ there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $|x_{n+1} - x_n| < \varepsilon$

(B) Given $\varepsilon > 0$ there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $\frac{1}{(n+1)^2}|x_{n+1} - x_n| < \varepsilon$

(C) Given $\varepsilon > 0$ there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $(n+1)^2|x_{n+1} - x_n| < \varepsilon$

(D) Given $\varepsilon > 0$ there exists an $n_0 \in \mathbb{N}$ such that for all m, n with $m > n \geq n_0$, $|x_m - x_n| < \varepsilon$

Q.4

Let $\vec{F}$ be a vector field given by $\vec{F}(x,y,z) = -y\,\hat{\imath} + 2\,x\,y\,\hat{\jmath} + z^3\hat{k}$, for $(x,y,z) \in \mathbb{R}^3$. If C is the curve of intersection of the surfaces $x^2 + y^2 = 1$ and $y + z = 2$, then which of the following is (are) equal to $\left|\int_C \vec{F} \cdot d\vec{r}\right|$?

(A) $\int_0^{2\pi}\int_0^1 (1 + 2r\sin\theta) r\, dr d\theta$

(B) $\int_0^{2\pi} \left(\frac{1}{2} + \frac{2}{3}\sin\theta\right) d\theta$

(C) $\int_0^{2\pi}\int_0^1 (1 + 2r\sin\theta)\ dr d\theta$

(D) $\int_0^{2\pi} (1 + \sin\theta)\, d\theta$

Q.5

Let V be the set of 2×2 matrices $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ with complex entries such that $a_{11} + a_{22} = 0$. Let W be the set of matrices in V with $a_{12} + \overline{a_{21}} = 0$. Then, under usual matrix addition and scalar multiplication, which of the following is (are) true?

(A) V is a vector space over $\mathbb{C}$
(B) W is a vector space over $\mathbb{C}$
(C) V is a vector space over $\mathbb{R}$
(D) W is a vector space over $\mathbb{R}$

Q.6

The initial value problem

$$y' = \sqrt{y},\ \ y(0) = \alpha,\ \ \alpha \geq 0$$

has

(A) at least two solutions if $\alpha = 0$

(B) no solution if $\alpha > 0$

(C) at least one solution if $\alpha > 0$

(D) a unique solution if $\alpha = 0$

Q.7 Which of the following statements is (are) true on the interval $\left(0, \frac{\pi}{2}\right)$?

(A) $\cos x < \cos(\sin x)$

(B) $\tan x < x$

(C) $\sqrt{1+x} < 1 + \frac{x}{2} - \frac{x^2}{8}$

(D) $\frac{1-x^2}{2} < \ln(2+x)$

Q.8 Let $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} xy\dfrac{x^2 - y^2}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

At $(0,0)$,

(A) f is not continuous

(B) f is continuous, and both f_x and f_y exist

(C) f is differentiable

(D) f_x and f_y exist but f is not differentiable

Q.9

Let $f, g : [0,1] \to [0,1]$ be functions. Let $R(f)$ and $R(g)$ be the ranges of f and g, respectively. Which of the following statements is (are) true?

(A) If $f(x) \leq g(x)$ for all $x \in [0,1]$, then $\sup R(f) \leq \inf R(g)$

(B) If $f(x) \leq g(x)$ for some $x \in [0,1]$, then $\inf R(f) \leq \sup R(g)$

(C) If $f(x) \leq g(y)$ for some $x, y \in [0,1]$, then $\inf R(f) \leq \sup R(g)$

(D) If $f(x) \leq g(y)$ for all $x, y \in [0,1]$, then $\sup R(f) \leq \inf R(g)$

Q.10

Let $f: (-1,1) \longrightarrow \mathbb{R}$ be the function defined by

$$f(x) = x^2 e^{1/(1-x^2)}$$

Then

(A) f is decreasing in $(-1,0)$

(B) f is increasing in $(0,1)$

(C) $f(x) = 1$ has two solutions in $(-1,1)$

(D) $f(x) = 1$ has no solutions in $(-1,1)$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 1 – Q. 10 carry one mark each.

Q.1

Let C be the straight line segment from $P(0,\ \pi)$ to $Q\left(4,\ \frac{\pi}{2}\right)$, in the xy-plane. Then the value of $\int_C\ e^x(\cos y\, dx - \sin y\, dy)$ is _______________

Q.2

Let S be the portion of the surface $z = \sqrt{16 - x^2}$ bounded by the planes $x = 0,\ x = 2,\ y = 0,$ and $y = 3.$ The surface area of S, correct upto three decimal places, is _______________

Q.3

The number of distinct normal subgroups of S_3 is _______________

Q.4

Let $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \left(1 + \frac{x}{y}\right)^2, & y \neq 0 \\ 0, & y = 0 \end{cases}$$

If the directional derivative of f at $(0,0)$ exists along the direction $\cos\alpha\ \hat{\imath} + \sin\alpha\ \hat{\jmath}$, where $\sin\alpha \neq 0,$ then the value of $\cot\alpha$ is _______________

Q.5

Let $f: \mathbb{R}^3 \longrightarrow \mathbb{R}$ be defined by

$$f(x,y,z) = \sin x + 2\, e^{\frac{y}{2}} + z^2$$

The maximum rate of change of f at $\left(\frac{\pi}{4},\ 0,\ 1\right)$, correct upto three decimal places, is _______________

Q.6

If the power series

$$\sum_{n=0}^{\infty} \frac{n!}{n^n} x^{2n}$$

converges for $|x| < c$ and diverges for $|x| > c$, then the value of c, correct upto three decimal places, is _______________

Q.7

If $5^{2015} \equiv n\ (\bmod\ 11)$ and $n \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then n is equal to __________

Q.8

If the set $\left\{ \begin{bmatrix} x & -x \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ x & x \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \right\}$ is linearly dependent in the vector space of all 2×2 matrices with real entries, then x is equal to _______________

Q.9

Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^6 - 1, & x \in \mathbb{Q} \\ 1 - x^6, & x \notin \mathbb{Q} \end{cases}$$

The number of points at which f is continuous, is _______________

Q.10

Let $f: (0,1) \to \mathbb{R}$ be a continuously differentiable function such that f' has finitely many zeros in $(0,1)$ and f' changes sign at exactly two of these points. Then for any $y \in \mathbb{R}$, the maximum number of solutions to $f(x) = y$ in $(0,1)$ is _______________

Q. 11 – Q. 20 carry two marks each.

Q.11 Let R be the planar region bounded by the lines $x = 0$, $y = 0$ and the curve $x^2 + y^2 = 4$, in the first quadrant. Let C be the boundary of R, oriented counter-clockwise. Then the value of

$$\oint_C x(1-y)\, dx + (x^2 - y^2)\, dy$$

is _______________

Q.12 Suppose G is a cyclic group and $\sigma, \tau \in G$ are such that order(σ) = 12 and order(τ) = 21. Then the order of the smallest group containing σ and τ is _______________

Q.13 The limit

$$\lim_{n\to\infty} \sum_{k=2}^{n} \frac{1}{k^3 - k}$$

is equal to _______________

Q.14 Let $M_2(\mathbb{R})$ be the vector space of 2×2 real matrices. Let V be a subspace of $M_2(\mathbb{R})$ defined by

$$V = \left\{ A \in M_2(\mathbb{R}) : A \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} A \right\}$$

Then the dimension of V is _______________

Q.15

Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{y}{\sin y}, & y \neq 0 \\ 1, & y = 0 \end{cases}$$

Then the integral

$$\frac{1}{\pi^2} \int_{x=0}^{1} \int_{y=\sin^{-1} x}^{\frac{\pi}{2}} f(x,y)\, dy\, dx$$

correct upto three decimal places, is ______________

Q.16

The coefficient of $\left(x - \frac{\pi}{4}\right)^3$ in the Taylor series expansion of the function

$$f(x) = 3 \sin x \cos\left(x + \frac{\pi}{4}\right), \qquad x \in \mathbb{R}$$

about the point $\frac{\pi}{4}$, correct upto three decimal places, is ______________

Q.17

If $\int_0^x \left(e^{-t^2} + \cos t\right)\, dt$ has the power series expansion $\sum_{n=1}^{\infty} a_n x^n$, then a_5, correct upto three decimal places, is equal to ______________

Q.18

Let ℓ be the length of the portion of the curve $x = x(y)$ between the lines $y = 1$ and $y = 3$, where $x(y)$ satisfies

$$\frac{dx}{dy} = \frac{\sqrt{1 + y^2 + y^4}}{y}, \qquad x(1) = 0$$

The value of ℓ, correct upto three decimal places, is ______________

Q.19 The limit

$$\lim_{x \to 0+} \frac{9}{x} \left(\frac{1}{\tan^{-1} x} - \frac{1}{x} \right)$$

is equal to ______________

Q.20 Let P and Q be two real matrices of size 4×6 and 5×4, respectively. If $\operatorname{rank}(Q) = 4$ and $\operatorname{rank}(QP) = 2$, then $\operatorname{rank}(P)$ is equal to ______________

END OF THE QUESTION PAPER